Self-Similar Solutions of Unsteady Mixed Convection Flow near the Stagnation Point of a Heated Vertical Plate in a Porous Medium Saturated with a Nanofluid

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CHAPTER ONE
INTRODUCTION

1.1 Background

This research aims to study the problem of unsteady mixed convection flow in a stagnation point near a heated vertical surface embedded in a nanofluid-saturated porous medium. The employed mathematical model for the nanofluid takes into account the effects of Brownian motion and thermophoresis. The presence of a solid matrix, which exerts first and second resistance parameters, is considered in this study. The self-similar solutions for the system of equations governing the problem are obtained. The resulting system of ordinary differential equations that governs the flow is solved numerically using fourth-fifth order Runge-Kutta with shooting method. The computations are done using the commercial packages NAG FORTRAN library and MATLAB. Numerical results for the dimensionless velocity, temperature and nanoparticle volume fraction as well as skin friction, Nusselt number and Sherwood number are produced for different values of the influence parameters.

1.2 Overview of the Research

The research contains four chapters, Chapter 2 discusses the governing equations and the mathematical modeling. Chapter 3 explains the results and the discussion of these results, and Chapter 4 contains the conclusion and the future work.

1.3 Literature Review

Fluid flow and heat transfer in porous media received considerable interest during the last decades. This is primarily because of the numerous applications in many engineering and geophysical fields, such as storage of radioactive nuclear waste, transpiration cooling, separation processes in chemical industries, filtration, transport processes in aquifers, ground water pollution, geothermal energy recovery and flow through filtering media.

During the past decade, the study of nanofluids has attracted enormous interest from research due to its exceptional applications to electronics, automotive, communication, computing technologies, optical devices, lasers, high-power X-rays, scientific measurement, material processing, medicine and material synthesis, where efficient heat dissipation is necessary. Nanobiotechnology is also a fast
developing field of research and application in many domains, such as in medicine, pharmacy, cosmetics and agro-industry. Nanofluids are prepared by dispersing solid nanoparticles in base fluids such as water, oil or ethylene glycol.

For regular fluid in clear media, the mixed convection flow in the stagnation flow region of a vertical plate has been investigated by Ramachandra et al. [1]. When there is an impulsive change in the velocity field, the inviscid flow develops instantaneously, but the flow in the viscous layer near the wall develops slowly which becomes fully-developed steady flow after sometime. For a small period of time, the flow is dominated by the viscous forces and the unsteady acceleration, but for a large period of time, it is dominated by the viscous forces, the pressure gradient and the convective acceleration. Scshadri et al. [2] have studied the unsteady mixed convection flow in the stagnation region of a heated vertical plate due to impulsive motion. The boundary-layer flow development of a viscous fluid on a semi-infinite flat plate due to impulsive motion of the free stream was investigated by Hall [3], Dennis [4] and Watkins [5]. The corresponding problem over a wedge was studied by Smith [6], Nanbu [7], and Williams and Rhyne [8]. Kumari [9] examined the temporal development of momentum and thermal boundary-layers on an impulsively-started wedge with a magnetic field and has obtained the solution numerically starting from the initial steady state to the final steady state. The flow development of the laminar boundary on an impulsively-started translating and spinning rotational symmetric body was considered by Ece [10]. Qzturk and Ece [11] studied the unsteady forced convection heat transfer from a translating and spinning body. Brown and Riley [12] presented an analysis that covered three distinct phases in the temporal development of the free convection flow past a suddenly heated semi-infinite vertical plate. The unsteadiness in the flow field arises due to the step change in wall temperature. Ingham [13] considered essentially the same problem as that of Brown and Riley [12], but instead of taking the step change in wall temperature, the wall temperature \( T_w \) is suddenly raised to \( T_w = T_w^* + Ax^m \), where \( A \) is a positive constant, \( m \) is a constant and \( x \) is the distance measured from the leading edge of the plate. The mixed convection flow at a two-dimensional stagnation point was investigated by Amin and Riley [14]. The forced flow is a stagnation point flow and the free convection part is due to a pressure gradient that is induced by temperature variations along the boundary. Fang et al. [15] introduced an analysis to the boundary-layers of an unsteady incompressible stagnation-point flow with mass transfer. Hydromagnetic stagnation point flow of a viscous fluid over a stretching or shrinking sheet was considered by Van Gorder et al. [16]. Mushtaq et al. [17] discussed the effect of thermal radiation on the stagnation-point flow of upper convected
Maxwell fluid over a stretching sheet. Hsiao [18] introduced a study to conjugate heat transfer for mixed convection and Maxwell fluid on a stagnation point. Recently Abel et al. [19] introduced numerical study of MHD boundary-layer stagnation point flow and heat transfer over an exponentially stretching surface with the Raman radiation. The problem of MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration was discussed by Mabood et al. [20].

For regular fluids in porous media, Hassanien et al. [21] analyzed the problem of unsteady free convection flow in the stagnation-point region of a rotating sphere embedded in a porous medium. The unsteady flow and heat transfer of a viscous fluid in the stagnation region of a three-dimensional body embedded in a porous medium was investigated by Hassanien et al. [22]. Hassanien and Al-Arabi [23] studied the problem of thermal radiation and variable viscosity effects on unsteady mixed convection flow in the stagnation region on a vertical surface embedded in a porous medium with surface heat flux. Rashidi and Erfani [24] introduced a new analytical study of MHD stagnation-point flow in porous media with heat transfer. Also, analytical solution of MHD stagnation-point flow in porous media by means of the homotopy perturbation method was introduced by Yildirim and Sezer [25]. Mabood and Khan [26] introduced an approximate analytic solutions for influence of heat transfer on MHD stagnation point flow in porous medium. Collocation solution of MHD stagnation point flow in porous media with heat transfer was introduced by Mohammadi and Rashidi [27]. Rosali et al. [28] discussed the effect of unsteadiness on mixed convection boundary-layer stagnation-point flow over a vertical flat surface embedded in a porous medium.

The research topic of nanofluids has received considerable interest worldwide. Inherently low thermal conductivity is a primary limitation in developing energy-efficient heat transfer fluids that are required for ultrahigh-performance cooling. A very small amount of guest nanoparticles, when dispersed uniformly and suspended stably in host fluids, can provide dramatic improvements in the thermal properties of the host fluids. According to Yacob et al. [29], nanofluids are produced by dispersing the nanometer-scale solid particles into base liquids with low thermal conductivity such as water and ethylene glycol. Nanoparticles are usually made of metal, metal oxide, carbide, nitride and even immiscible nano-scale liquid droplets. Congedo et al. [30] compared different models of nanofluid (regarded as a single phase) to investigate the density, specific heat, viscosity and thermal conductivity and discussed the water–Al2O3 nanofluid in details by using CFD. Hamad et al. [31] introduced a one-parameter group to represent similarity reductions for the problem of magnetic field effects on free-
convective nanofluid flow past a semi-infinite vertical flat plate following a nanofluid model proposed by Buongiorno [32]. Hady et al. [33] obtained the analytical solutions for convective flow and heat transfer of a viscous incompressible nanofluid past a semi-infinite vertical stretching sheet in the presence of magnetic field. Hady et al. [34] discussed the effect of suction/injection on natural convective boundary-layer flow of a nanofluid past a vertical porous plate through a porous medium. Further, Abu-Nada and Chamkha [35] presented the natural convection heat transfer characteristics in a differentially-heated enclosure filled with a CuO–EG–water nanofluid for different variable thermal conductivity and variable viscosity models. The unsteady flow and heat transfer of a nanofluid over a contracting cylinder studied by Zaimi et al. [36]. Srinivasacharya and Surender [37] studied the effects of thermal and mass stratification on natural convection boundary-layer flow over a vertical plate embedded in a porous medium saturated by a nanofluid. For more informations see also Das et al. [38] and Kakaç and Pramuanjaroenkij [39]. Muthtamilselvan et al. [40] claimed that it is difficult to have a precise idea on how nanoparticles enhance the heat transfer characteristics of nanofluids. Mustafa et al. [41] analyzed the problem of stagnation-point flow of a nanofluid towards a stretching sheet. Makinde et al. [42] studied buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet. The problem of axisymmetric stagnation flow of nanofluid in a moving cylinder was analyzed by Nadeem and Rehman [43]. Tamim et al. [44] investigated the steady laminar MHD mixed convection boundary-layer flow of a nanofluid near the stagnation point on a vertical permeable plate with prescribed external flow and surface temperature. Das [45] introduced Lie group analysis of stagnation-point flow of a nanofluid.

In this study, our main objective is to analyze mixed convection flow in stagnation point of a heated vertical surface embedded in a nanofluid-saturated porous medium. The effect of the presence of an isotropic solid matrix due to impulsive motion is considered. Moreover, we examine the combined effect of Brownian motion, thermophoresis parameters and nanoparticle fraction on boundary-layer flow and heat transfer due to nanofluid. The governing boundary-layer equations are transformed to a two-point boundary-value problem using similarity variables. These are numerically solved using fourth-fifth order Runge–Kutta method with shooting technique. The computations are done using the numerical packages NAG FORTRAN library and MATLAB. The effects of governing parameters on fluid velocity, temperature and particle concentration are discussed and results are shown graphically and in tabular form.
الملخص العربي

يهدف البحث إلى دراسة مسألة الحمل الحراري المختلط غير المستقر للسريان بالقرب من نقطة الركود لسطح عمودي مسخن ومغموس في وسط مسامي مشبع بسائل نانو. النموذج الرياضي المستخدم لوصف السائل النانو يأخذ في الاعتبار تأثيرات الحركة البراونية والترحيل الحراري (الثيرموفورثيث). في هذه الدراسة سنعتبر وجود المصفوفة الصلبة التي تحتوي بارامترات المقاومة الأولى والثانية. تم الحصول على الحلول شبه الذاتية لنظام المعادلات التي تحكم المسألة. كما تم حل نظام المعادلات التفاضلية العادية التي تحكم التدفق عددياً باستخدام طريقة رونج كوتا من الدرجة الرابعة. الخامسة مع طريقة الشوتنج. واستخدمت حزمة البرامج العددية (NAG) مع كود فورتران وبرنامج MATLAB في الحصول على النتائج العددية التي توضح توزيعات السرعة ودرجة الحرارة والجسيمات النانوية فضلاً عن الاحتكاك السطحي وعدد نسبي وعدد شرويد لقيم مختلفة من البامترات التي تحكم المسألة.
ABSTRACT

This work aims to study the problem of unsteady mixed convection flow in a stagnation point near a heated vertical surface embedded in a nanofluid-saturated porous medium. The employed mathematical model for the nanofluid takes into account the effects of Brownian motion and thermophoresis. The presence of a solid matrix, which exerts first and second resistance parameters, is considered in this study. The self-similar solutions for the system of equations governing the problem are obtained. The resulting system of ordinary differential equations that governs the flow is solved numerically using fourth-fifth order Runge-Kutta with shooting method. The computations are done using the commercial packages NAG FORTRAN library and MATLAB. Numerical results for the dimensionless velocity, temperature and nanoparticle volume fraction as well as skin friction, Nusselt number and Sherwood number are produced for different values of the influence parameters.
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